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PHYSICAL AND MATHEMATIC SCIENCE SOLVING HIGHER DEGREE EQUATIONS

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Abstract
This article presents some methods for solving higher order equations.

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The fundamental theorem of algebra:

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0 \qquad (1)$$

although it implies that there are n solutions to the equation, in general it does not specify algorithms for finding solutions. Solving equation (1) has been a difficult problem until now, and it has been solved only in some special cases.

If the solution of the equation (1) is determined by the expression formed by performing the operations of addition, subtraction, multiplication, division and extraction of roots on the coefficients of the equation, then the equation (1) is said to be solved in radicals.

Reminder. If $\alpha = a + bi$ complex number is a solution of equation (1), α number joint $\overline{\alpha} = a + bi$ complex number is also a solution of equation (1).

Now let's look at some cases where equation (1) is solved in radicals.

Obviously, $n = 2$ when equation (1) comes to the previously studied quadratic equation: $ax^2 + bx + c = 0$

This equation always has two roots:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \qquad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

(discriminant $b^2 - 4ac > 0$ when, x_1 and x_2 are real and distinct numbers; $b^2 - 4ac = 0$ when $x_1 = x_2$ is a multiple of the root; $b^2 - 4ac < 0$ when x_1 and x_2 are mutually exclusive complex numbers).

Now we consider the following special cases of equation (1).

a) $n = 3$ let it be In these cases, equation (1).

$$a_0 x^3 + a_1 x^2 + a_2 x + a_3 = 0$$

appears. For convenience, the key equation is as follows

$$a_0 x^3 + a_1 x^2 + a_2 x + a_3 = 0 \qquad (a_0 \neq 0) \qquad (2)$$

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we can record. Equation (2) is solved as follows:
`1) Dividing both sides of equation (2) by , we find:

$$x^3 + b_1x^2 + b_2x + b_3 = 0 \qquad (3)$$

In this $\quad b_k = \frac{a_k}{a_0} \quad$ (k = 1,2,3)

1) (3) in Eq $\quad x = y - \frac{b_1}{3}$

we will replace. Then the left side of equation (3) is this

$$\left(y - \frac{b_1}{3}\right)^3 + b_1\left(y - \frac{b_1}{3}\right)^2 + b_2\left(y - \frac{b_1}{3}\right)^3 + b_3 =$$

$$y^3 + \left(b_2 - \frac{b_1^{\ 2}}{3}\right)y + \left(b_2 - \frac{b_1b_2}{3} + \frac{2}{27}b_1^{\ 3}\right)$$

it looks like the following

$$\left(b_2 - \frac{b_1^{\ 2}}{3}\right) = p, \qquad \left(b_2 - \frac{b_1b_2}{3} + \frac{2}{27}b_1^{\ 3}\right) = q$$

equation (3) after substitution

$$y^3 + py + q = 0 \qquad (4)$$

Takes the from.

2) (4) the solution of the equation

$$y = u + v \qquad (5)$$

we look for in the form, where u and v are these

$$u \cdot v = -\frac{p}{3} \qquad (6)$$

is required to satisfy the condition.
Evidently, u and v considered from relations (5) and (6) are as follows

$$t^2 - yt - \frac{p}{3} = 0$$

it follows that the roots of the quadratic equation (Viet's theorem).

3) $y = u + v$ ni (4) we substitute y in the equation:

$$(u+v)^3 + p(u+v) + q = 0$$

As a result

$$u^3 + 3u^2v + 3uv^2 + v^3 + pu + pv + q = 0$$

ie

$$\left(u^3 + v^3 + q\right) + (3uv + p)(u+v) = 0$$

(7)

Will be.

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Known, $u \cdot v = -\frac{p}{3}$. In it $3uv + p = 0$ is, equation (7).

$u^3 + v^3 + q = 0$, ie $u^3 + v^3 = -q$

appears. So,

$y^3 + py + q = 0$
solving this equation

$$\begin{cases} u^3 + v^3 = -q \\ u^3 \cdot v^3 = -\frac{p^3}{27} \end{cases} \qquad (8)$$

comes to solve the system.

4) It can be seen from equations (8) that u^3 and v^3 s these

$t^2 + qt - \frac{p^3}{27} = 0$

will be the solutions of the quadratic equation. We solve this quadratic equation:

$t_1 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$, $t_1 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$

So,

$$u^3 = t_2 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \qquad (9)$$

$$v^3 = t_2 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \qquad (10)$$

6) (9) and (10) from the equations

$$\begin{matrix} u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \\ v = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \end{matrix} \qquad (11)$$

we find that Hence, the solution of equation (4).

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$$y = u + v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \qquad (12)$$

Will be.

(12) the equality is called Cardano's formula. This is the formula

$$u+v$$

consists of a sum, and each u and v has three values. Then the values of u+v sum will be 9. Among these values, only three are solutions of equation (4), and the values of such u and v are as follows

$$uv = -\frac{p}{3}$$

will relate.

7) Suppose u and v are one of the values of equations (11).

u_1 and v_1 let it be In it

$$u_2 = \frac{-1+\sqrt{3}\cdot i}{2}u_1, \qquad u_3 = \frac{-1-\sqrt{3}\cdot i}{2}u_1,$$

$$v_2 = \frac{-1-\sqrt{3}\cdot i}{2}v_1, \qquad v_3 = \frac{-1+\sqrt{3}\cdot i}{2}v_1$$

Will be.

7) (4) solutions of Eq

$$y_1 = u_1 + v_1$$

$$y_2 = -\frac{1}{2}(u_1+v_1) + \frac{\sqrt{3}\cdot i}{2}(u_1 - v_1) \qquad (13)$$

$$y_3 = -\frac{1}{2}(u_1+v_1) - \frac{\sqrt{3}\cdot i}{2}(u_1 - v_1)$$

are the solutions of the given equation

$$x_1 = y_1 - \frac{b_1}{3}, \qquad x_2 = y_2 - \frac{b_1}{3}, \qquad x_3 = y_3 - \frac{b_1}{3}$$

Will be.

1-misol. *This,*

$$x^3 - 9x^2 + 21x - 5 = 0$$

solve the equation.

◄ In the given equation

$$x = y + 3$$

we will replace. In it

$$(y+3)^3 - 9(y+3)^2 + 21(y+3) - 5 = 0$$

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ie

$$y^3 - 6y + 4 = 0$$

will be. (12) we find using the formula.

$$u = \sqrt[3]{-2+\sqrt{4-8}} = \sqrt[3]{-2+2i}$$

Obviously, this is one of the values of the cube root

$$u_1 = 1 + i$$

Will be. In it

$$v_1 = \frac{6}{3(1+i)} = 1 - i$$

is, according to formula (13).

$$y_1 = 2, \qquad y_2 = -1-\sqrt{3}, \qquad y_3 = -1+\sqrt{3}$$

will be. Hence, the solutions of the given equation

$$x_1 = 5, \qquad x_2 = 2-\sqrt{3}, \qquad x_3 = 2+\sqrt{3}$$

Will be. ▶

a) $n = 4$ let it be In this case, equation (1).

$$a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \qquad (14)$$

appears. Equation (14) is solved in radicals. It has a solution algorithm.

REFERENCES

1. O`taganova Umida Egamberdi qizi. General Formula for Calculation of dimensions and Surfaces of Some Geometric Figures. In recognition of the paper publication of the research paper on International Journal on Orange Technoligies. 2022.6.18
2. O`taganova Umida Egamberdi qizi. Planimetry as a Structural Component of Structure for Professional training of a Future teacher of Mathematics. In recognition of the paper publication of the research paper on International Journal on Orange Technoligies. 2022.6.4
3. O`taganova Umida Egamberdi qizi. Qo`zg`aluvchan massalar yordamida rotor harakatini balanslash. 2023.5.30
4. O`taganova Umida Egamberdi qizi. Research Medhods of Mathematics Teaching Methodology. 2023.5.12
5. Turayev, H., Raimkulov, S. U., & O'taganova, U. E. qizi . (2024). GEOMETRIK MASALALARNI YECHISHDA IZLASH METODIDAN FOYDALANISH HAQIDA. Educational research in universal sciences, 3(4), 48–52. https://doi.org/10.5281/zenodo.10920812

6. Jumayev Bekzod Zokirovich, Jo‘rayev Ilhom Ro‘ziboy o‘g‘li “ОБЩАЯ ТЕОРИЯ ВЫЧИСЛЕНИЯ ЦИФР ЧИСЛА, КОТОРОЕ ПОЛУЧАЕТСЯ, ЕСЛИ МЫ ВОЗВОДИМ ЧИСЛО 7 ДО ПРОИЗВОЛЬНОГО n-ГО ПОРЯДКА” JOURNAL OF NEW CENTURY INNOVATIONS. Volume–43_Issue-4_December_2023. 34-37 betlar. http://www.newjournal.org/
7. O‘taganova Umida Egamberdi qizi, Jo‘rayev Ilhom Ro‘ziboy o‘g‘li “ФОРМУЛЫ ДЛЯ ВЫЧИСЛЕНИЯ КВАДРАТА ЛЮБЫХ НАТУРАЛЬНЫХ ЧИСЕЛ” Educational Research in Universal Sciences. VOLUME 2 | SPECIAL ISSUE 18 | 2023. December 126-129 betlar.
8. Jo‘rayev Ilhom Ro‘ziboy o‘g‘li “OXIRGI RAQAMI 6, 7 VA 8 BO‘LGAN SONLARNING KVADRATINI HISOBLASH FORMULALARI”. SCHOLAR. VOLUME 1 | ISSUE 29 | 2023. October. 93-98 betlar.